Exercise 92

(a) Use a CAS to differentiate the function

$$f(x) = \sqrt{\frac{x^4 - x + 1}{x^4 + x + 1}}$$

and to simplify the result.

(b) Where does the graph of f have horizontal tangents?

(c) Graph f and f' on the same screen. Are the graphs consistent with your answer to part (b)?

Solution

Use Mathematica to differentiate the function and simplify the result.

$$f'(x) = \frac{-1 + 3x^4}{(1 + x + x^4)^2 \sqrt{1 - \frac{2x}{1+x+x^4}}}$$

The graph of f has horizontal tangents where $f'(x) = 0$.

$$\frac{-1 + 3x^4}{(1 + x + x^4)^2 \sqrt{1 - \frac{2x}{1+x+x^4}}} = 0$$

This can occur only when the numerator is zero.

$$-1 + 3x^4 = 0$$

Solve for x.

$$x = \pm \frac{1}{\sqrt[4]{3}} \approx \pm 0.759836$$

Below is a plot of $f(x)$ and $f'(x)$ versus x.